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ON THE ELECTRICAL HEATING OF WIRES IN RAREFIED GASES BY AN ELECTRIC CURRENT

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16. Abstract

This article outlines experimental work to determine the temperature dependence of the heat loss of thin wires in hydrogen.

An explanation for the peculiar form of the current-voltage curve has not previously appeared in literature.

This 46 page study ends with four conclusions, supported by 71 equations and 25 Figures.

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Introduction

§ 1. If the current-voltage curve of an iron wire stretched /401* through rarefied hydrogen is recorded under the proper conditions, a peculiar phenomenon appears, which is shown schematically in Fig. 1. With increasing voltage e the current i increases at first, although at lower rate than the voltage. However, from a certain voltage value onwards, the current increase ceases (Point a). From there onwards, the current remains almost constant over an appreciable range of voltage until, after a certain increment ab (Point b), it rather suddenly begins to climb again.

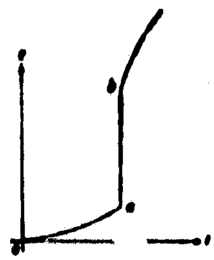

Fig. 1.

This property of such iron resistances has been known for a long time and is commonly used in technological applications in order to keep the current in an apparatus independent of the line voltage, as in Nernst lamps in particular. Such resistances are also used (under the name "Variators") as starting resistances for smaller motors.

An explanation for the peculiar form of the current-voltage curve has not previously appeared in the literature; the well-known high temperature coefficient of resistance for iron has been invoked now and then. While this explains the strong curvature of the lower branch 0-a of the characteristic, the origin of the vertical slope of branch a-b remains to be clarified.

* Numbers in the margins indicate page number in original foreign text

It will be sought in the following to give this clarification in the frame of a general investigation of the behavior of electrically heated wires in rarefied gases.

We will begin our task with the heat balance equation:

(1) $$i^2 R(T) = A(T),$$

(i = current,

T = temperature of the wire,

$R(T)$ = resistance of the wire at temperature T,

and $A(T)$ = heat loss per second at the temperature T).

When R and A are known as functions of the temperature, this equation gives the temperature for each current i, as well as the electrical resistance R and the terminal voltage $e = i R$.

Numerous measurements of the temperature dependence of the electrical resistance are available in the literature, so that the function $R(T)$ can be considered to be known for almost all metals that might be considered. The temperature dependence of the heat loss A requires, on the other hand, a more penetrating investigation, to which the first part of the following work is directed. We will use these results in the second part for construction of characteristics; we will find that in the case of just the iron resistance discussed above, the characteristic constructed on the basis of Eq. (1) cannot be realized because it corresponds to a labile state. The real behavior of wires will be derived, and a theory of these resistances will be given. Finally in the third part the results of the theory will be tested against experiment.

I. The Laws of Cooling

§ 2. Three phenomena are in general effective in cooling a heated body: radiation, heat conduction, and convection.

a) The radiation is given by the Stefan-Boltzmann law, which reads as follows for a black object:

$$A_s(T) = \sigma (T^4 - T_o^4) \cdot o. \tag{2}$$ /403

where T = the absolute temperature of the object,

T_o = the absolute temperature of the surroundings,

o = the surface area of the object,

and σ = the radiation constant = 5.76×10^{-12} Watt/cm^2-deg^2 [1]

With an object not perfectly black, the absorptivity appears as a factor on the right hand side. This quantity in general depends on the temperature. With polished metals the radiation law is modified by a dependence of the absorptivity on the wavelength of the radiation as well. The theory (Aschkinass[2]) gives in this case a radiation formula which we can write in the following form with the use of the radiation constant:

$$A_s = 1{,}04 \cdot \sqrt{s} \left[\left(\frac{T}{1000} \right)^{4\frac{1}{2}} - \left(\frac{T_0}{1000} \right)^{4\frac{1}{2}} \right] \cdot o \text{ Watt}, \tag{3}$$

where s represents the specific resistance of the metal at the temperature T in Ω-mm^2/m, i.e. the resistance of a wire of 1 m length and 1mm^2 in cross section.

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[1] W. Gerlach, Zeitschr. f. phys. 2 p. 76, 1920.

[2] E. Aschkinass, Ann. d. phys. 17 p. 960, 1905.

The fraction of the heat loss due to radiation is appreciable only at high temperatures and very small pressures; we will consider it in general to be correction factor.

b) While the radiation losses simply add to the remaining heat losses, this is not true of the heat lost by <u>convection</u>. These losses cannot be determined separately; rather, the process of heat conduction is modified by convection. The heat current carried with the moving gas alters the temperature distribution in the gas and thus, the heat loss through conduction as well. For this reason, convection is not very accessible either to experimental or theoretical study. To the author's knowledge, the only available attempt at a theory is that of L. Lorenz[3]; for a flat vertical plate suspended in open space he obtains the /404
following expression for the quantity of heat lost per unit surface area:

$$A_k = C \cdot l^{-1/4} \rho^{1/2} (T - T_0)^{5/4}, \tag{4}$$

with C = a constant,

l = the height of the plate,

and ρ = the density of the surrounding gas at a large distance from the plate.

This formula can obviously apply only in those cases in which the heat loss is large in comparison with the gas at rest. Without this assumption, the heat loss per degree of temperature excess $A_K/(T-T_0)$, calculated on the basis of the formula, decreases beneath all limits with increasing l or decreasing $(T-T_0)$ and would ultimately become smaller than the heat loss in the gas at

[3] L. Lorenz, Wied. Ann. 13, p. 579, 1884.

rest, which is impossible. Thus the Lorenz result, as confirmed by experiment, is of very limited application.

We will therefore restrict ourselves in the following to those cases in which the heat loss through convection is excluded as much as possible. According to Kundt and Warburg[4] this can always be achieved by making the gas pressure sufficiently small, for on the one hand, this decreases the buoyancy (the driving force of the gas motion) thus reducing the gas velocity, while on the other hand the heat capacity of a unit volume will also decrease, decreasing the cooling effect of the gas. This pressure dependence is expressed in the Lorenz formula (4) by the factor $\rho^{\frac{1}{4}}$.

c) Thus we are left with heat conduction as the essential cause of heat loss through the surrounding gas. From now on, we will consider a definite arrangement; a wire of radius r_1 and length l is stretched along the axis of a cylindrical vessel of radius r_o; the absolute temperature of the wire is T_1; that of the vessel is T_o. The wire length we take to be so large in comparison with its radius that we can consider the wire to be infinitely long; i.e., the temperature along the entire length of the wire can be considered to be the same. Thus we ignore heat conduction from the ends of the wire. Let χ be the thermal conductivity of the gas and T the temperature at a distance r from the axis. Consider a cylindrical shell of radius r and thickness dr concentric with the wire. In the stationary state the same heat current passes through each such shell, namely, the quantity of /405

[4] A. Kundt and E. Worburg, Pogg. Ann. 156, p. 177, 1875.

heat given off from the wire:

$$A_L = -2\pi r \kappa \frac{dT}{dr} l.$$

Now according to the kinetic theory of gases, χ is proportional to $\sqrt{T}$ in agreement with the average of the rather scattered experimental results.[5] Let us therefore set

$$\kappa = \kappa_0 \sqrt{T},$$

thus

$$A_L \frac{dr}{r} = -2\pi\kappa_0 l \sqrt{T}\, dT,$$

and integrated

(5)
$$A_L = \frac{\frac{4}{3}\pi\kappa_0 l (T_1^{3/2} - T_0^{3/2})}{\ln\frac{r_0}{r_1}}.$$

This gives the desired temperature dependence of the heat loss by conduction. According to this, A_L is independent of gas pressure.

§ 3. Formula (5) is, however, valid only at high gas pressures. At lower pressures, the heat transport is affected by the temperature jump occurring between the hot surface of the wire and the gas layer immediately neighboring the surface. For this case the formula of Smoluchowski[6] applies for the heat loss:

(5a)
$$A_L = \frac{2\pi\kappa l (T_1 - T_0)}{\ln\frac{r_0}{r_1} + \gamma\left(\frac{1}{r_1} + \frac{1}{r_0}\right)},$$

[5] Landolt-Bornstein, Phys. - Chem. Tab., 4th Ed., p. 743, 1912.

[6] M. V. Smoluchowski, Wied. Ann. 64, p. 101, 1898.

where γ is the "temperature jump coefficient", i.e., that length which gives the temperature jump when multiplied by the temperature gradient in the gas. Here γ is of the order of magnitude of the mean free path length L in the gas and is proportional to it, /406 and is thus inversely proportional to the gas pressure p. It is easily seen that with decreasing pressure p, the second term of the denominator grows; thus A in fact decreases. At very large pressures the second term disappears in comparison with the first; then the Smoluchowski formula goes over into our formula (5)-at least for small temperature differences($T_1-T_0 \ll T_0$) for which one can put $3/2\sqrt{T_0}(T_1-T_0)$ in place of $(T_1^{3/2}-T_0^{3/2})$.

In fact the Smoluchowski formula is valid only for small temperature differences, since χ as well as γ were assumed to be independent of T in its derivation. In reality χ and γ vary appreciably with temperature; χ, as remarked above is proportional to $\sqrt{T}$, and γ is proportional to the mean free path length L. Thus γ is, say, $\gamma_0 L = \gamma_0 L_0 T$, since L is proportional to the absolute temperature. Since, according to the theoretical ideas[7] of the origin of the temperature jump, an appreciable variation of the proportionality constant γ_0 with temperature is not to be expected, it may be assumed

[7] cf. for example:

M. Knudsen, Ann. d. Phys. 34, p. 593, 1911.
M. V. Smoluchowski, Ann. d. Phys. 35, p. 983, 1911.
H. Bolza, M. Born and Th. V. Karman, Gottinger Nachr., 1913.
B. Baule, Ann. d. Phys. 44, p. 145, 1914.

that γ is proportional to T. Because of its neglect of this temperature dependence, the Smoluchowski formula is unsuited for our purposes, where temperature differences up to almost 1000° are to be assumed. The question now arises whether the derivation of this formula can be adapted to large temperature differences. This is in fact possible. However the formula obtained is so complicated that it is suitable neither for obtaining a general understanding of the problem, nor for technical applications. Above all, the calculation can be carried through, as in the Smoluchowski case, only under the assumption that the mean free path is small in comparison with the wire radius. For wire radii of the same order of magnitude as L or smaller, there is as yet no theory. This assumption, however, means in our case, in which the wire radius is small in comparison with that of the surrounding vessel, that the term $\gamma\left(\frac{1}{r_1} + \frac{1}{r_0}\right)$ /407
in Eq. (5a) is small in comparison with $\ln \frac{r_0}{r_1}$, i.e., that the influence of the temperature jump on the heat loss is small. Thus, if the temperature jump appreciably affects the heat loss, the assumption above cannot be fulfilled.[8]

At low gas pressures we are, therefore, forced to rely on experiment to obtain the heat loss.

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[8]With Smoluchowski, the situation was different. He considered the diameter of the inner heat-emitting cylinder to be only slightly smaller than that of the surrounding vessel; thus $\ln r_0/r_1$ was of appreciably smaller order. Thus, in his case, the temperature jump exerted a noticeable effect on the quantity of heat transported from $\frac{L}{r_1} \approx \frac{1}{100}$ on.

§4. The experiments were set up so that a measured electric current i could be sent through the wire under study and the terminal voltage e could be determined between its ends. Then in the stationary state the quantity of heat lost, expressed in energy units, is equal to the electrical energy input ei. The temperature, on the other hand, can be determined from the resistance $R=e/i$, if this is known as a function of the temperature.

Fig. 2 shows the experimental apparatus. The test wire D was stretched vertically through a thin walled glass vessel G, of 30 mm diameter and provided with a cooling jacket. The ends of the wire were clamped tightly in 3mm thick brass rods. For ease in changing the wire, the upper rod was cemented into a slide S, while the lower rod was only a short piece to which a copper wire was soldered whose lower end was immeresed in mercury, which provided a path for the current. In this fashion the wire could be

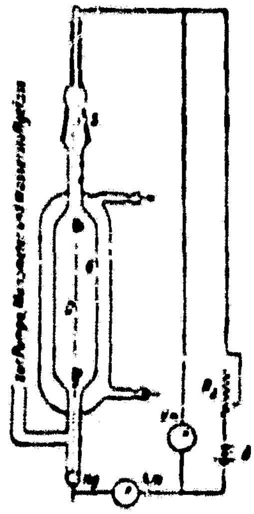

Fig. 2

stretched evenly and could respond to the rather considerable thermal expansion at the high temperatures employed. Regulation of the temperature of the walls of the vessel was provided by water flowing through the cooling jacket. /408
The water temperature was determined with a mercury thermometer. The vessel G could be evacuated with a Gaede rotary mercury pump. The pressure measurement at higher pressures was made with a mercury manometer; at lower pressures (below 4mm Hg) the measurement was made with a McLeod manometer. The hydrogen used to fill the vessel was prepared electrolytically from potash lye with nickel electrodes and dried with calcium chloride and phosphorus pentoxide. Control measurements which were carried out with purest hydrogen, which had been diffused in through a red-hot palladium tube, resulted in the same values, indicating that the electrolytically prepared hydrogen was sufficiently pure. Platinum was chosen as the material of the

wire, since on the one hand it is best suited to determination of the temperature and, on the other hand, affords a clean well-defined surface. So called "hair wires"[9] of 0.0513 mm diameter were used. The wire length amounted to 10 cm overall. Current and voltage measurements were carried out with precision laboratory instruments (Ammeter Am and voltmeter Vm). The voltage drop across the ammeter was calculated from the ammeter resistance and was used to correct the measured voltage.

§ 5. Sources of Error and Corrections. The following sources of error come into question: 1. The thermal resistance of the glass wall, and 2. the finite wire length (heat loss through the leads). One can imagine the effect of the first replaced by hydrogen layer of the same thermal resistance. Thus it amounts to an apparent enlargement of the radius of the vessel. Since, however, glass conducts heat about five times better than hydrogen and the thickness of the glass only amounted to about 0.5 mm, this enlargement of the vessel radius amounts to only about 0.1 mm and can, therefore, be neglected. On the other hand, the heat loss at the ends of the wire has an important effect, which appears as follows. Because of the lower temperature of the ends both the resistance (and therewith the temperature) as well as the heat loss appear smaller than would be the case with a wire at a uniform temperature. One can eliminate this error by measuring the voltage, not between the ends of the wire, but rather between /409 two points sufficiently separated from the ends. This method is, however, not applicable to our thin wires, since the leads necessary for measurement of the voltage, however thin one may attempt to make them, must exert a cooling effect and lower the temperature of the wire at the point of attachment. Another way would be to send the same current through two wires of different length stretched in the same fashion. The heat loss and temperature might then

[9] Supplier: Hartmann und Braun, A.-G., Frankfurt a.M.

be determined from the difference in the measured voltages.[10] However, because of the yet unclear effect of the convection, which by Eq. (4) must depend upon the wire length, the author has not chosen to use this method, but has preferred to calculate the required correction, which was determined in the following fashion.

Let χ be the distance of a point on the wire from an end; λ, the thermal conductivity of the metal of the wire in Watt/cm-deg; q, the cross section; and $A^{O}(T)$ and $R^{O}(T)$, the heat loss and resistance per centimeter of wire length, respectively. Then the temperature distribution along the wire will be determined through the differential equation

$$\lambda q \frac{d^2 T}{d x^2} = A^0(T) - i^2 R^0(T) \tag{6}$$

with the accessory condition that the temperature of the end of the wire $(x = x_0)$ is equal to the temperature of the surroundings. This is an assumption, which can be regarded as fulfilled in view of the large cross section of the brass rods providing lead connections. We may further assume that the wire is so long that the temperature in the middle is independent of x $\left(\frac{dT}{dx} = \frac{d^2T}{dx^2} = 0\right)$. We can then regard the wire as infinitely long on one side and do not need to worry about the other end. The equation can be easily integrated in the case that $A^{O}(T)$ and $R^{O}(T)$ are linear functions of T: /410

$$A^0(T) = C\vartheta; \quad R^0(T) = R_1^0(1 + a\vartheta);$$
$$\vartheta = T - T_0$$

(R_1^{O} = resistance per cm at temperature T_0. C and a = constants). Then we have

$$\frac{d^2\vartheta}{dx^2} = \frac{1}{\lambda q}\{C\vartheta - i^2 R_1^0 a\vartheta - i^2 R_1^0\} = \beta^2(\vartheta - \vartheta_\infty).$$

[10] R. Goldschmidt, Physik. Zeitschr. 12, p. 417, 1911.

with

$$\vartheta_m = \frac{i^2 R_0'}{C - i^2 R_0' \alpha}; \quad \beta^2 = \frac{C - i^2 R_0' \alpha}{\lambda q} = \frac{i^2 R_0'}{\lambda q \vartheta_m}.$$

ϑ_m is the temperature at the middle of the wire, since $d^2\vartheta/dx^2$ =0 for $\vartheta = \vartheta_m$.

Taking into account the accessory condition, the solution of the differential equation becomes:

$$\vartheta = \vartheta_m (1 - e^{-\beta x})$$

or

(7)
$$\begin{cases} T = T_m - (T_m - T_0) e^{-\beta x}, \\ \beta = i \sqrt{\dfrac{R_0'}{\lambda q (T_m - T_0)}}. \end{cases}$$

The temperature measured in our set up is the average value

$$T_i = \frac{2}{l} \int_0^{l/2} T \, dx = T_m - (T_m - T_0) \frac{2}{\beta l},$$

from which the correction formula for the temperature follows:

(8)
$$\begin{cases} T_x = T_i + \dfrac{(T_x - T_0)^{3/2}}{i} \cdot K \\ \text{mit} \quad K = 2 \sqrt{\dfrac{\lambda q}{l R(t_0)}}. \end{cases}$$

For our experimental set up (platinum wire of 0.0513 mm diameter, 10 cm long, R=5.14 Ohm at 0^0 , λ =0.70 Watt/cm-deg) the factor K is calculated to be
$$K = 1{,}06 \cdot 10^{-3} \frac{\text{Amp.}}{\sqrt{\text{deg}}} = 1{,}06 \frac{mA}{\sqrt{\text{deg}}}.$$

The observed heat loss is then
$$A_b = 2 \int_0^{l/2} A^0(T) \, dx = 2 A^0(T_x) \left[\frac{l}{2} - \frac{1}{\beta} \right] = A(T_x) \left[1 - \frac{2}{\beta l} \right];$$

Thus the following correction formula results for A: /411

(9)
$$A(T_x) = \frac{A_b}{1 - \dfrac{\sqrt{T_x - T_0}}{i} K},$$

or approximately (A_b/i = e = terminal voltage):

(9a) $$A(T_\infty) = A_b\left(1 + \frac{1}{i}\sqrt{T_\infty - T_0}\,K\right) = A_b + e\sqrt{T_\infty - T_0}\cdot K.$$

Earlier we assumed a linear variation for the functions A(T) and R(T). If we now drop this assumption, then the solution of the differential equation can no longer be brought into a useable form. Nevertheless we can give an approximate solution if A and R do not deviate strongly from linearity. For T we assume a function of the form of (7), but we give the constant β a different value, namely one that allows dT/dx to take the proper value at x=0. This value can rigorously be calculated from the differential equation (6); upon taking into account the subsidiary condition dT/dx= 0 for $T=T_\infty$:

$$\left(\frac{dT}{dx}\right)^2 = \frac{2}{\lambda q}\left\{-\int_T^{T_\infty} A^0(T)\,dT + i^2\int_T^{T_\infty} R^0(T)\,dT\right\},$$

thus

$$\left(\frac{dT}{dx}\right)_{x=0} = \sqrt{\frac{2}{\lambda q}\left\{i^2\int_{T_0}^{T_\infty} R^0(T)\,dT - \int_{T_0}^{T_\infty} A^0(T)\,dT\right\}}$$

Since on the other hand our assumption of (7) gives

$$\left(\frac{dT}{dx}\right)_{x=0} = \beta(T_\infty - T_0)$$

one finds after some rearrangement the following value for

$$\beta = i\sqrt{\frac{R_0}{\lambda q(T_\infty - T_0)}}\,\frac{1}{f(T_\infty)}$$

with

(10) $$f(T_\infty) = \frac{1}{\sqrt{\frac{2}{(T_\infty - T_0)R(T_0)}\int_{T_0}^{T_\infty} R(T)\,dT - \frac{R(T_\infty)}{R(T_0)}\,\frac{2}{(T_\infty - T_0)A(T_\infty)}\int_{T_0}^{T_\infty} A(T)\,dT}}.$$

/412

and thus the following correction formula for the temperature is found.

(L)

$$T_r = T_b + \frac{(T_r - T_b)}{l} K \cdot f(T_r).$$

For the measurements the average value of the resistance R appearing in Eq. (1) was calculated from the constants of the resistance formula, while A was in general determined by graphical integration; f proved to be always not far from one, differing by at most 20% in the most extreme case.

The correction formula for the heat loss could not be developed in general form. The observed heat loss

$$A_b = 2\int_0^{l} A^0(T)\, dx$$

was, therefore, calculated under the assumption of various functions $A^0(T)$, some of which rose more rapidly, others more slowly, than in a linear fashion. Always there resulted for A_b an expression of the form

$$A_b = A(T_\infty)\left[1 - \frac{2}{\beta l} g(T_\infty)\right];$$

The factor $g(T_\infty)$ was always almost equal to the reciprocal value of $f(T_\infty)$, so that the product fg, appearing as a factor multiplying K in formulae (9) and (9a), can to a satisfactory approximation be set equal to one. These formula, therefore, maintain their validity even for the case of a nonlinear variation of A(t) and R(T).

§ 6. Results of Measurements. First the resistance function of the platinum wire material employed was determined. To this end the wire was wound in the usual fashion on a mica cross. This was sealed into a glass vessel filled with rarefied hydrogen and the resistance thermometer thus prepared was compared with a platinum resistance thermometer of Heraeus that had been calibrated in a boiling apparatus. The comparison was made in a paraffin oil bath at temperatures between 0° and $380^{\circ}C$ in steps of about 20°. The results could be represented to within 1/10 degree by the quadratic

equation

(11) $$\frac{R}{R_0} = 1 + 0{,}003241\, t - 0{,}449 \cdot 10^{-6}\, t^2$$

(t= the temperature in Celsius degrees, R_0= resistance at 0^0C.) Without hesitation this formula could be used to this high degree of accuracy even for higher temperatures (up to about 900^0), in spite of the low purity of the metal shown by the low value of the first constant. With the actual measurements the determination of the average wire temperature from the measured resistance was made with the aid of the above equation using graphical methods. /413

In this fashion current and voltage were determined for the sample wire. From them the temperature and heat loss were calculated as described in § 4 and were corrected according to formulae (8a) and (9a). The results are summarized in the curves of Figs. 3 and 4, where the heat loss is plotted as a function of the temperature for various gas pressures p. /414

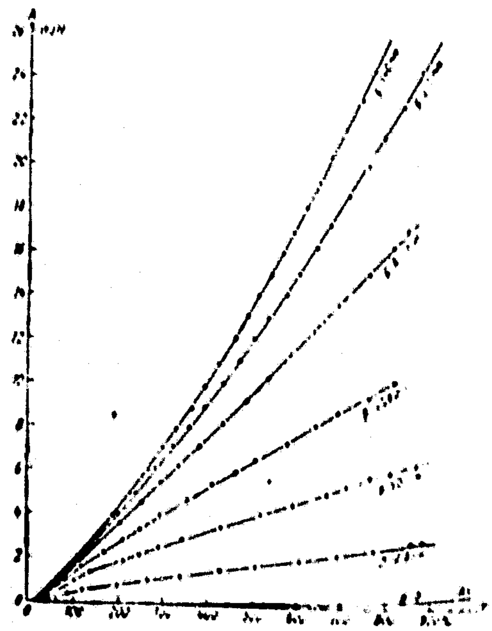

Fig. 3

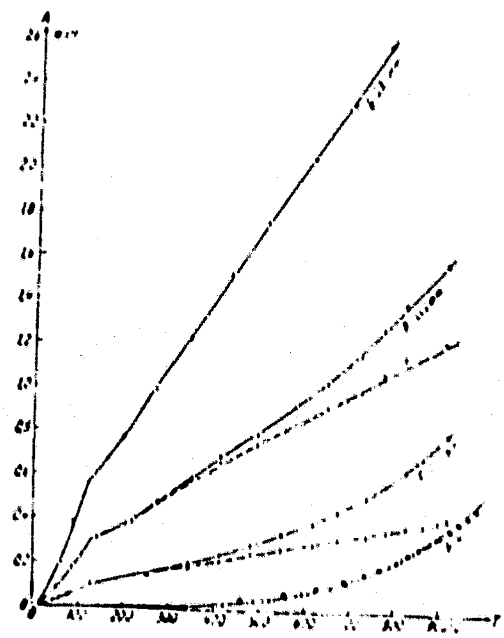

Fig. 4

The heat loss A thus measured is composed of radiation as well as conduction losses, which are eventually affected by convection. In order to be able to ascertain the conduction contribution by itself, the radiation loss was determined by a special measurement at the highest vacuum; the gas pressure was no more than 0.0001 mm higher than the mercury vapor pressure, which amounted to about 0.001 mm. Here the application of the corrections formulae (8a) and (9a) appeared to be no longer admissible, since the A(T) curve (Eq. 3) deviated very strongly from linearity. Therefore, A was determined by forming the difference of the measured voltages on two wires of different length (10.0 and 34.2 cm) carrying the same current. This result is given by the points marked with simple crosses and the curve labeled with p=0 in Fig. 4. The curve is computed according to the Aschkinass theory (Eq. 3); however, the calculated A values had to be multiplied by a factor of 1.7 in order to reach agreement with the measurements. Thus the absorptivity of the platinum surface had to be 1.7 times greater than the calculation assumes. The origin of this difference will not be further discussed here. At the same time, the opportunity was taken to test correction formulae (8a) and (9a). The A values calculated with the aid of these formulae applied to the shorter wire of 10 cm in length are marked in Fig. 4 with the circled crosses. On account of the wire length, these were the most unfavorable conditions for the use of these formulae. It is obvious that the deviation from the correct values is small, particularly at the higher temperatures, in spite of the large size of the correction, which at times amounted to as much as 40% of the measured value. The formulae thus lead to approximately correct results even far outside of the domain for which they were derived.

It is clear from Fig. 4 that the contribution of radiation to the heat loss becomes considerable only at the lowest pressures. For these pressures

the remaining heat loss (by conduction) after subtraction of the radiation is given by the two dashed curves in Fig. 4.

§ 7. Before we undertake the task of calculating the current-voltage curve /415 from the measured variation of the function A(T), there remains to be discussed the form of the A-curves and the conclusions to be drawn from them in relation to the heat conduction process in hydrogen. At high pressure the curves are convex towards the abscissa and follow fairly exactly the law

$$A(T) = \mathrm{const}\cdot(T^{1,4} - T_0^{1,4})$$

as Fig. 3 shows, where the curve passed through the second series of points[11] from the top (p=250 mm) is calculated by this law. With decreasing pressure A decreases, and in fact, more rapidly the higher the temperature is; the curvature decreases, and near p=80 mm the curves are almost straight lines. By further decreases in the pressure they show a concave curvature towards the abscissa, which constantly increases with decreasing pressure after subtraction of the radiation (dashed curves.) Without this correction, but as a consequence of the radiation, the concavity towards the abscissa later decreases again and finally changes over to convex curvature towards the abscissa. The variation pictured here is, however, only relevant at higher temperatures. At lower temperatures the behavior is quite different. Here the curves are almost exactly linear, and what is most striking the transition to the higher temperature region results in a pronounced cusp, particularly at the lowest pressures.

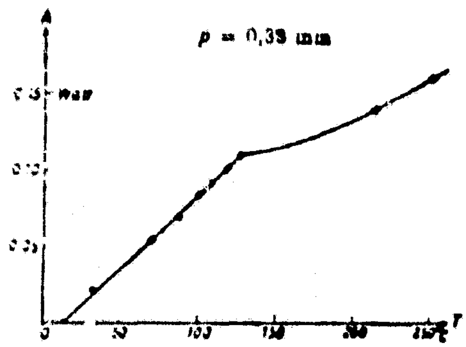

Fig. 5.

[11] The top series of points (p=740mm) were not considered because of their somewhat anomalous behavior, obviously a consequence of the effect of convection (see below).

In Fig. 5, which shows the lower portion of the curve for p=0.33 mm on an enlarged scale, this cusp is particularly clear. Recording the points directly above the cusp was difficult, because the lability phenomena discussed below appeared. This is a proof that to the left of the cusp, the curve must show a very shallow rise. At every pressure the cusp lies at about the same temperature, namely around 120°. Further experimental series showed that the cusp moves to higher temperatures, both with increasing temperature T_0 of the surroundings and with decreasing wire diameter. /416

The origin of this striking separation between two different regions of temperature is very difficult to understand. The first idea that might occur is that convection is significant and that the cusp represents the transition from laminar flow to turbulence. This idea must be rejected since the Reynolds number here is too small by many orders of magnitude. The clarification of this question, which is far afield from the purpose of this work anyway, must therefore await later investigations.

We will now attempt to represent the observations above 200° by a formula. The structure of the Smoluchowski formula suggests that we should try an equation of the type of our formula (5). The denominator of the right side would then have to be expanded with a contribution dependent on the "temperature jump". Therefore the expression

$$F(T) = \frac{T^{1/2} - T_0^{1/2}}{\Delta i}$$

was calculated from the measurements and was plotted in Figs. 6 and 7. Fig. 7 gives the F values for the smaller pressures. For graphical reasons the product pF was chosen as ordinate. For the bottom curve, which otherwise would fall on top of the next one above it, the scale of the ordinate was reduced in the ratio 3:4. It is clear that each series of points falls with great accuracy on a straight line. Only the curve obtained at atmospheric pressure is noticeably curved. Since it is reasonable to assume that convection played

a disturbing role here, we will not consider these points in the following. The deviation of the highest temperature points on the lines corresponding to the lower pressures is probably connected with a nonuniform temperature distribution caused by lability (cf. Part II), which was at times observed here and which would have to throw the measurements into error. /417

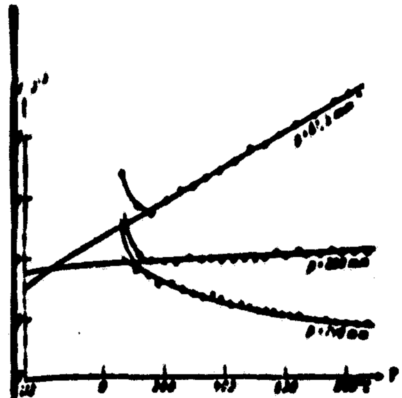

Fig. 6.

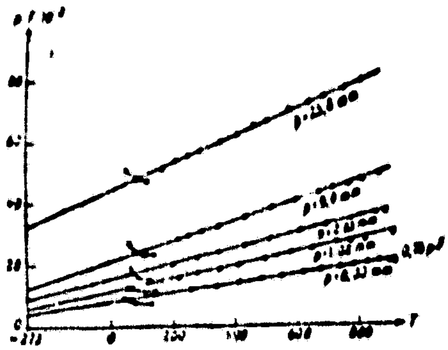

Fig. 7.

Thus F can be represented by the equation

$$F(T) = M + N\,T$$

and A_L by

(12) $$A_L = \frac{T'^{\,\cdot} - T_0'^{\,\cdot}}{M + N\,T},$$

where M and N are constants which depend on the gas pressure p and can be determined from Figs. 6 and 7. They are tabulated in the third and fourth columns of Tab. I for the different pressures.

Table I[a]

p mm Hg	$\frac{L_0}{r_1}$	$10^{-1} M$ obs	N obs	$10^{-1} M \cdot p$ obs	$N \cdot p$ obs	$10^{-1} M$ cal	N cal	$10^{3} n$ cal
250	0,0222	1,35	0,057	337,5	14,3	1,80	0,221	0,170
81,5	0,0671	1,30	0,626	106	51	1,80	0,626	0.481
25,9	0.212	1,30	1,67	33,5	43	1,31	1,67	1,275
9,0	0,61	1,50	3,52	13,5	32	1,55	3,55	2,29
2.85	1,92	3,23	8,60	9,2	24,5	2,99	8,46	2,83
1,02	5,36	6,3	20,6	6,4	21	7,0	20,4	2,92
0,33	16,6	19	55	6,3	19	20,2	62	3,04

27

In the second column the ratio L_0/r_1 of the mean free path length in the gas at the temperature T_0 (= 10^0C) to the wire radius is given. It is obvious that M is almost constant and N is approximately proportional to p so long as L_0/r_1 is small in comparison with one. Conversely, if L_0/r_1 is large in comparison with one, then the products Mp and Np are approximately constant, as columns 5 and 6 show. This last product has, however, a smaller value than at high pressure. Only the values at the highest pressure (p=250 mm) deviate. Here there is probably, as at atmospheric pressure, a disturbing influence exerted by convection. /418

In order to determine more exactly the dependence of the quantities M and N on p, their reciprocals were plotted as functions of p. This is carried out in Fig. 8, which shows that 1/M and 1/n can most simply be represented by exponential functions. Thus one obtains from M and N the empirical formulae:

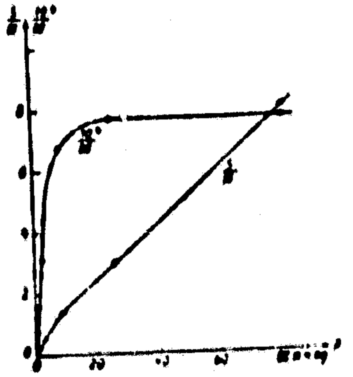

Fig. 8.

$$M = \frac{1300}{1 - e^{-1,1\, r_1/L_0}}; \tag{13a}$$

$$N = \frac{10,2}{\frac{r_1}{L_0} + 1,4\left(1 - e^{-1,1\, r_1/L_0}\right)} \tag{13b}$$

The values of M and N calculated with these formulae are reproduced in columns 7. and 8. of Table 1. They show satisfactory agreement with the quantities determined from observation.

For higher pressures at which the exponential terms can be neglected ($p > 20$ mm), these formulae yield the following simplified formula for the heat loss:

$$A_L = \frac{T - T_0}{1300 + 10,2 \cdot T_0 \frac{L_0\, T\, T_0}{r_1 + 1,4\, L_0}} \tag{14}$$

When one considers that L_0T/T_0 is the mean free path length in the immediate vicinity of the wire, then one notices the strong similarity with the Smoluchowski equation (5a), extended to larger temperature differences by analogy with Eq. (5): /419

(5b)
$$A_L = \frac{\frac{4}{3}\pi\varkappa_0 l(T^{3/2} - T_0^{3/2})}{\ln\frac{r_2}{r_1} + \gamma_0\frac{L}{r_1}},$$

Our formula differs from the above in only one respect: in the second term of the denominator the radius r_1 a-pears increased by a contribution equal to 1.4 times the mean free path. Equating the two formulae yields for and the values

$$\varkappa_0 = \frac{\ln\frac{r_2}{r_1}}{\frac{4}{3}\pi l\cdot 1300} = 1{,}17\cdot 10^{-4}\frac{\text{Watt}}{\text{cm deg}^{3/2}}$$
$$\gamma_0 = \frac{\ln\frac{r_2}{r_1}\cdot 10{,}2\cdot T_0}{1300} = 14{,}2$$

and for the thermal conductivity of hydrogen at 0°C.

$\chi(0^{\circ}) = \chi_0\sqrt{273} = 1.93 \times 10^{-3}$ Watt/cm-deg $= 4.62 \times 10^{-4}$g cal/cm-deg-sec

$\chi(100^{\circ}) = \chi_0\sqrt{373} = 2.26 \times 10^{-3}$ Watt/cm-deg $= 5.40 \times 10^{-4}$g cal/cm-deg-sec

Other authors have measured earlier the following values:

$\chi(0^{\circ}) = 3.19 \times 10^{-4}$ (Graetz[12]),

$\chi(0^{\circ}) = 3.27 \times 10^{-4}$ (Winkelmann[13]),

$\gamma_0 = 6.96$ (Smoluchowski[14] for hydrogen on glass),

$\gamma_0 = 5.70$ (Gehrcke[15] for hydrogen on silver),

$\gamma_0 = 5.6$ (Smoluchowski[16] for hydrogen on platinum from the measurements of Knudsen)

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12 L. Graetz, Wied. Ann. 14, p. 232, 1881.
13 A. Winkelmann, Wied. Ann. 44, pp. 177 & 429, 1891.
14 M. V. Smoluchowski, loc. cit.
15 E. Gehrcke, Ann. d. Phys. 2, p. 103, 1900.
16 M. Smoluchowski, Ann. d. Phys. 35, p. 983, 1911.

The comparison shows that the agreement of our empirical formula (14) with (5b) is only formal. In particular our formula yields a value of the temperature jump coefficient more than two times too large. In this connection, agreement is not to be expected, since the use of formula (5b) at larger temperature differences can not be justified theoretically. Thus one is not justified in drawing conclusions about the temperature dependence of the temperature jump coefficient from the observed formal similarities. /420

On the other hand, there are difficulties in explaining the large deviation of the thermal conductivity from the values of other observers. It is noteworthy that Schleiermacher[17], who determined the thermal conductivity of hydrogen by a method similar to that used here, although on much thicker wires, found values of the thermal conductivity that were also too large, namely:

$$\varkappa(0^\circ) = 4{,}10 \cdot 10^{-4},$$
$$\varkappa(100^\circ) = 5{,}23 \cdot 10^{-4},$$

The last value deviates only slightly from our result. The process of heat conduction through gases in the vicinity of thin wires thus requires clarification by further experiments, in which the wire radius,[18] in particular, should be varied.

II. The Current-Voltage Curves and the Dependence of the Wire Temperature on the Current.

§ 8. By means of the function A(T) determined in Part I, it is now possible for us to construct the current-voltage curve for wires of arbitrary material using the method sketched in the introduction. The required additional function R(T) giving the dependence of the wire resistance on temperature, we take from the work of Somerville [19], who investigated the temperature coefficient of resistance of a larger number of different metals

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[17] A. Schleiermacher, Wied. Ann. 34, p. 623, 1888. cf. also S. Weber Ann. d. Phys. 54, pp. 325 & 437, 1917, and references there cited.

[18] It should be especially emphasized that formulae (13a) and (13b) do not pretend to represent the dependence of the heat loss on the wire radius. The quantity r_1 is introduced into them only for the sake of a more transparent formulation.

[19] A. Somerville, Phys. Rev. 31, p. 261, 1910.

over a wide temperature range (0-1000°). In the interest of simplicity, we will, unless expressly stated otherwise, always assume /421 in the following that the wire length is so large that the influence of the ends need not be considered. Thus we assume that the temperature can be considered to be uniform over the entire length of the wire.

The construction of the current-voltage curve can be carried out graphically without calculation as follows: The connection among the voltage e, current i, heat loss per sec. A and resistance R is given through the equations

$$e \cdot i = A(T),$$
$$\frac{e}{i} = R(T);$$

or logarithmically

$$\log e + \log i = \log A(T),$$
$$\log e - \log i = \log R(T).$$

In a coordinate system in which log i is drawn as the abscissa and log e serves as ordinate, the lines log A=constant are parallel straight lines inclined at 45° to the coordinate axes. The lines log R=const. are likewise parallel straight lines which are perpendicular to the first group. This means, however, that every point in the log e-log i coordinate system is systematically a point in the rectangular log R- log A coordinate system which is rotated 45° counterclockwise with respect to the log e-log i system, as Figure 9 shows. In addition, the scale in the log R-log A system is smaller by a factor $1/\sqrt{2}$ than in the log e-log i system. In order to find the e-i curve one needs only to plot the R-A curve (which is parametrically represented by the functions A(T) and R(T)) on logarithmic graph paper. The e-i curve is then found by rotating the coordinate axes by 45° clockwise. If a temperature scale is placed on the R-axis next to the resistance scale, then all relevant quantities can be read off the diagram. /422

Fig. 9.

This construction is carried out in Figure 10 for an iron wire of 10 cm length and 0.0513 diameter using the A-curves obtained in Part I and the Somerville resistance values. The surprising result is found that the e(i) curves after rising monotonically from small values of the current and voltage.

bend backwards; i.e., that the current, which at first had risen with the voltage, reaches a maximum and decreases with further increases in voltage. At still higher voltage the current reaches a minimum and then increases again with increasing voltage. One may take the abscissa difference between maximum and minimum in the logarithmic representation, that is, the ratio of the currents at these points as a measure for the size of the effect. The effect is most pronounced at medium pressures (p=9 mm); it decreases at higher and lower pressures and is almost imperceptible at atmospheric pressure.

We wish to use the name "reversing characteristic" for those current-voltage curves for which the transition from the rising section to the decreasing section occurs through a <u>vertical</u> section that is, there is a maximum of the current. This is in contrast to the "decreasing" curves for which the transition occurs through a horizontal section (a voltage maximum).

Before we turn ourselves to the question of the real occurrence of such reversing characteristics, we want to determine the condition that the construction leads to curves of this type. The condition is obviously that the voltage, or rather the temperature (since the voltage always increases /423
with increasing temperature, as is evident in Fig. 10) increases with decreasing current i, i.e., that $dT/di < 0$. The derivative dT/di is calculated from Eq. (1):

$$2 \log i = \log A(T) - \log R(T),$$

$$\frac{2}{i}\frac{di}{dT} = \frac{1}{A}\frac{dA}{dT} - \frac{1}{R}\frac{dR}{dT}.$$

After we introduce the dimensionless quantities

$$\alpha = \frac{T}{A}\frac{dA}{dT}, \qquad \rho = \frac{T}{R}\frac{dR}{dT}$$

We have the following:

$$(15) \qquad \begin{cases} \dfrac{T}{i}\dfrac{di}{dT} = \dfrac{\alpha-\rho}{2}, \\[2ex] \dfrac{i}{T}\dfrac{dT}{di} = \dfrac{2}{\alpha-\rho}. \end{cases}$$

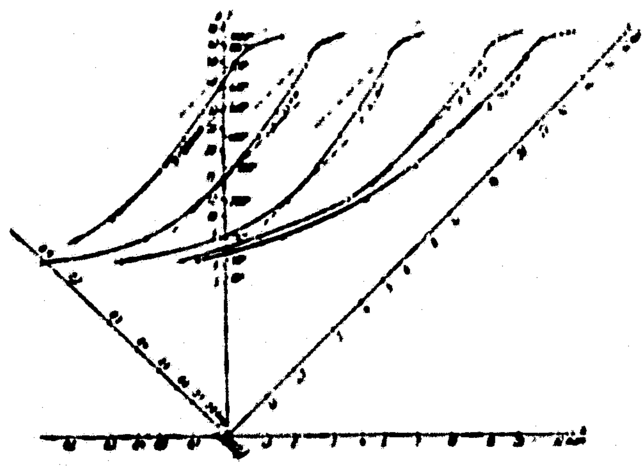

Fig. 10.

The quantities α and ρ, which are the logarithmic derivatives of the functions A(T) and R(T), are equal to the powers of the absolute temperature with which A and R vary in the vicinity of the temperature in question. Calculation of the corresponding quantities for the functions R(i) and e(i) yields:

$$(16) \qquad \begin{cases} \varphi = \frac{i}{R}\frac{dR}{di} = \frac{i}{R}\frac{dR}{dT}\frac{dT}{di} = \rho \cdot \frac{i}{T}\frac{dT}{di} = \frac{2\rho}{\alpha-\rho}; \\ \epsilon = \frac{i}{e}\frac{de}{di} = \frac{1}{R}\left(R + i\frac{dR}{di}\right) = 1 + \varphi = \frac{\alpha+\rho}{\alpha-\rho}. \end{cases}$$

The condition for the appearance of a reversing characteristic with de/di and dT/di < 0, respectively follows from Eq. (15 and (16), respectively:

$$(17) \qquad \alpha - \rho < 0,$$

i.e., the heat loss A must show a smaller percentage growth with temperature than the resistance R.

If α and ρ are plotted as functions of T, the condition (17) requires that the α-curve lie beneath the ρ-curve. Since this can only occur in a piecewise fashion, the condition requires that the two curves intersect. Such an intersection point ($\alpha - \rho = 0$) implies according to Eq. (16) that de/di = ∞. Thus it corresponds to a current maximum or minimum in the characteristic, as in Fig. 10. /424

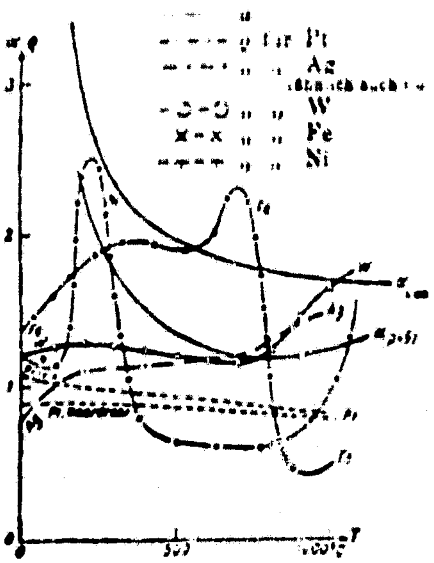

Fig. 11.

Fig. 11 shows the curves $\rho = \rho$ (T) for various metals. They are calculated from the above-mentioned data of Somerville on the temperature coefficient

$$a = \frac{1}{R_0}\frac{dR}{dT}$$

The curves were calculated by means of the equation

$$\rho = T\frac{R_0}{R}a,$$

except for the values for platinum. The values for the material of the "hair wire" samples were calculated from Eq. (11).

For pure platinum the following equation was employed:

$$\frac{R}{R_0} = 1 + 0{,}003945\, t - 0{,}584 \cdot 10^{-6}\, t^2$$

With Pt, Ag, Cu and W ρ lies in the vicinity of one, since with these metals the resistance grows almost proportionately to the absolute temperature. With iron and nickel ρ attains substantially higher values. The curve for iron possesses a shallow maximum at 300-400° upon which a peak at 700° is superimposed. The ρ-curve for Ni shows a sharp maximum at about 200°. For both metals the maximum values of ρ are larger than 2. Because of the strong effect of slight impurities as well as mechanical and thermal pre-treatment, the behavior of these metals can of course be described only in general terms. /425

The quantity α we calculate from the results of Part I and obtain the following results.

a) With cooling by radiation, we find from Eq. (3)

$$\alpha_S = \frac{T}{A_S}\frac{dA_S}{dT} = \frac{4{,}5\, T^{4,5}}{T^{4,5} - T_0^{4,5}} + \frac{1}{2}\frac{T}{s}\frac{ds}{dT},$$

Since

$$\frac{T}{s}\frac{ds}{dT} = \frac{T}{R}\frac{dR}{dT} = \rho \approx 1 \quad ,$$

we obtain the approximation

(18) $$\alpha_S = \frac{4{,}5}{1 - \left(\frac{T_0}{T}\right)^{4,5}} + \frac{1}{2}$$

For temperatures above 300°C, for which $\left[1 - \left(\frac{T_0}{T}\right)^{4,5}\right]$

is almost one, we have then

(18a) $$\alpha_S \approx 5;$$

b) With cooling only by conduction at high pressure we find from Eq. (5):

(19) $$\alpha_L = \frac{\frac{3}{2} T^{3/2}}{T^{3/2} - T_0^{3/2}} = \frac{3/2}{1 - \left(\frac{T_0}{T}\right)^{3/2}};$$

c) With cooling only by conduction at low pressure, we find from Eq. (12):

$$\text{(20)} \qquad \alpha_L = \frac{\frac{3}{2}T'}{T' - T_0'} - \frac{nT}{1+nT} = \alpha_{L\infty} - \frac{nT}{1+nT},$$

where n=N/M. The last column of Table I gives the values of n for different pressures.

d) With cooling by both conduction and radiation, we find:

$$\text{(21a)} \qquad \alpha = \frac{A_L}{A_L + A_S}\alpha_L + \frac{A_S}{A_L + A_S}\alpha_S$$

$$\text{(21b)} \qquad = \alpha_L + \frac{A_S}{A_L + A_S}(\alpha_S - \alpha_L).$$

The following should be noted in connection with the formulae: /426

a) With radiative cooling alone, as with wires in high vacuum, $\alpha = \alpha_S$ is always larger than 5 and is thus far larger than ρ . In this case (e.g. incandescent lamps) reversing characteristics can never appear.

b) $\alpha_{L\infty}$ (Conductive cooling at high pressure) is according to Eq. (19) always larger than 3/2. The curve $\alpha_{L\infty}(T)$ is drawn in on Fig. 11. It intersects only one of the ρ -curves, namely that of iron at about 550 and 770°. With iron wires in hydrogen at high pressure (about atmospheric pressure) reversing characteristics are therefore to be expected only in a limited temperature domain.

c) At lower pressures α_L is according to Eq. (20) always smaller than $\alpha_{L\infty}$ by the amount nT/(1+nT), which can assume a maximum value of 1, and which, besides, depends on the pressure p as well as the temperature. According to Table 1, n grows steadily with decreasing pressure and attains at zero pressure a finite limiting value of about 0.003; nT/(1+nT) varies similarly with p. We will refrain from reproducing the α_L-curves, because at low pressure the curves are strongly affected by radiation.

d) In the general case α is calculated from α_L and α_S using a kind of rule of mixtures (Eq. 21a). The form (21b) is more convenient. Since $\alpha_S - \alpha_L$ varies only inside narrow limits (in the temperature domain 200-1000°C, only between 2.7 and 4.1), this form shows that because of radiation a contribution is added to α_L. This contribution is proportional to the ratio of the radiative loss to the total heat loss; it is therefore

very small at low temperatures, but at higher temperatures and low pressures, it increases rapidly with the temperature. Thus it can occur that at low pressure and higher temperatures α increases with increasing temperature, although α_L and α_S each decrease as the temperature rises. The α -curves at lower pressure must, therefore, pass through a minimum at a certain temperature.

The α -curves thus calculated appear in Fig. 12. and confirm the discussion above. At higher pressures α decreases with increaing temperature. With decreasing pressure p, the α -curves at first slide downwards /427 because of the decrease of α_L. From p=9 mm onwards the curves begin to climb again at higher temperatures and slide upwards again by further decreases in pressure as a consequence of the increasing influence of the second term in Eq. (21b). The curve for p=9 mm lies lowest of all. It thus corresponds to the smallest α -values. Therefore Eq. (17) will be satisfied over the widest temperature domain, and the reversing characteristic will be the most pronounced, as the diagram of Fig. 10 already shows. This curve (p=9 mm) is also drawn in on Fig. 11. Besides the ρ -curve of iron, it also intersects the curves for Ni, Ag, and W with three points of intersection in the first case and one intersection point for each of the last two metals. Thus reversing characteristics are to be expected with these metals as well.

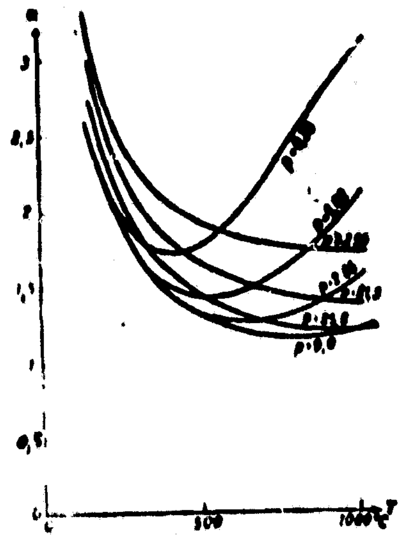

Fig. 12.

§ 9. Let us now turn ourselves to the question of the real occurrence of reversing characteristics. Such curves have never been observed, and even in the author's experiments-- to get ahead of the story-- they have not appeared. How can this difference between theory and reality be explained? Since, as is well known, unstable states can easily appear in conductors having a "decreasing" characteristic[20], it is plausible to seek the origin of

[20] W. Kaufmann, Ann. d. Phys. 2, p. 158, 1900; for a review, see for example, H. Busch, Stabilit, Labilitat und Pendelungen in der Electrotechik ("Stability, Lability and Oscillation in Electrotechnology"), Leipzig (S. Hirzel) 1913, ch. I.

the deviation in a lack of stability. In fact the Kaufmann stability criterion is usually not satisfied by a conductor having a reversing characteristic and connected to a current source through a series resistance R_v; for this /428
criterion demands that

$$R_v > -\frac{de}{di}.$$

In the vicinity of a maximum or minimum of the current where -de /di grows beyond all bounds, this can surely not be the case, however great the series resistance R_v may be.

Closer investigation shows, however, that the Kaufmann stability criterion is not applicable here. In this connection let us consider the diagram of the logarithmic characteristic in Fig. 13; e is the characteristic of the conductor, while E is that of the source of current with the series resistance included. The stationary state corresponds to the intersection point P of the two curves. Let us now assume that the equilibrium is disturbed by giving the conductor a somewhat smaller temperature than that corresponding to the stationary state. In this case its resistance will be somewhat smaller, so that the state corresponds, say, to the point P_1 on the characteristic E. In order for the state to be stable, the perturbation of the equilibrium must disappear by itself; that is, the temperature must increase. Thus the heat influx must be larger than the loss. Let us put through P_1 a line of equal resistance, i.e. equal temperature. This is the line $\overline{P_1Q}$ parallel to the A-axis which cuts the characteristic e at the point P_0. The heat influx is then given by the length $\overline{QP_1}$; the outflow, by $\overline{QP_0}$
Thus the stability condition is that the point P_1 lies rightwards and above P_0; i.e., that the characteristics E and e intersect the way they do in Fig. 13, or expressed analytically:

$$-\frac{dE}{di} < -\frac{de}{di}$$

In the special case that E is the voltage of a constant battery with a series resistance R_v ($E=E_0-iR_v$) the condition becomes

$$R_v < -\frac{de}{di}.$$ /429

The Kaufmann condition must not be exactly satisfied for the state to

be stable.[21] Thus labile states appear only when the characteristic of the current source is very steep, like, say, the dotted curve E^* in Fig. 13.

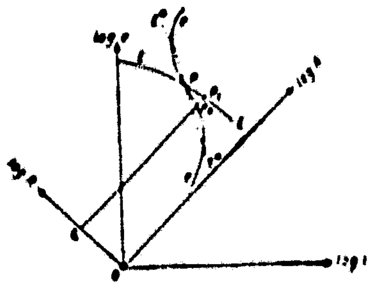

Fig. 13.

Let us investigate further the case of two equal resistances with reversing characteristics connected in series to a battery of constant voltage E_0. The condition for the equilibrium state is then

$$E_0 - e_1(i) = e_2(i),$$

where $e_1(i)$ and $e_2(i)$ are the voltages on the respective resistances. The possible equilibrium states are found by allowing the characteristic $e(i)$ to intersect the curve $e=E_0$, as in Fig. 14.

The figure (14) shows that three intersection points appear in general, of which the two outer ones S_1 and S_3 correspond to stable states. On the other hand, the stability appears doubtful in the state corresponding to the center intersection point S_2, since both resistances have equal temperatures and resistance and both have reversing characteristics. Here the stability must be specially investigated. In this case, however, the graphical method applied above is not applicable, because the state is determined by two mutually independent variables, namely the temperatures of the two resistances. We are therefore forced into an analytic treatment.

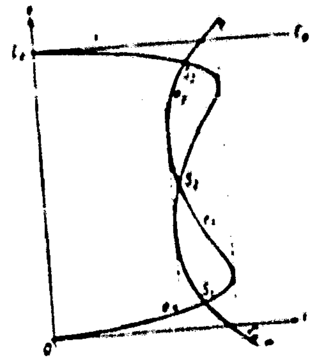

Fig. 14.

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[21]Essential for the result is the alteration of R and T in the same sense, i.e., a positive temperature coefficient for the conductor. As is easily seen, the inverse result appears with a negative temperature coefficient. Thus the Kaufmann criterion is again found. If the characteristic is determined by thermal processes, then conductors with "decreasing" characteristics, like those Kaufmann investiated, in fact, always have negative temperature coefficients of resistance.

This can be carried out without difficulty using the well-known method of small variations and well-defined rules of calculation. However, we wish to adopt a simpler method, which, although it does not supply us the complete stability condition, will give us the most important necessary condition. /430

Let the temperatures of the two resistances be T_1 and T_2, their heat capacity, K, and let t be the time. Then the two following equations are valid:

$$i^2 R(T_1) - A(T_1) = K \frac{dT_1}{dt},$$
$$i^2 R(T_2) - A(T_2) = K \frac{dT_2}{dt}.$$

In the stationary state at the point S_2 ($d/dt=0$, $T_1=T_2=T_0$) the following holds for both resistances:

$$i_0^2 R(T_0) - A(T_0) = 0.$$

Let us now consider a state deviating slightly from equilibrium. Thus we put

$$T_1 = T_0 + \vartheta_1, \quad T_2 = T_0 + \vartheta_2, \quad i = i_0 + j,$$

where θ_1, θ_2, and j are quantities so small that their squares and products can be neglected, and we can put

$$R(T_1) = R(T_0 + \vartheta_1) = R(T_0) + \vartheta_1 R'(T_0) \text{ mit } R'(T_0) = \left(\frac{dR}{dT}\right)_{T=T_0}$$

etc.

Then the following results:

$$R(T_0) 2 i_0 j + [i_0^2 R'(T_0) - A'(T_0)] \vartheta_1 = K \frac{d\vartheta_1}{dt},$$
$$R(T_0) 2 i_0 j + [i_0^2 R'(T_0) - A'(T_0)] \vartheta_2 = K \frac{d\vartheta_2}{dt}$$

and by forming the difference:

$$[i_0^2 R'(T_0) - A'(T_0)](\vartheta_1 - \vartheta_2) = K \frac{d(\vartheta_1 - \vartheta_2)}{dt}.$$

For stability it is necessary that deviations from the equilibrium state vanish by themselves, i.e., that the absolute values of θ_1 and θ_2 decrease with time. The same, however, must apply to the absolute value of the difference since this difference must likewise vanish in the stationary state. A necessary, but not sufficient, stability condition is thus that $\theta_1 - \theta_2$ and $d(\theta_1 - \theta_2)/dt$ have opposite signs. By the last equation this yields /431

the condition

$$i_0^2 R'(T_0) - A'(T_0) = \frac{i_0^2 R(T_0)}{T_0}\left[\rho(T_0) - \alpha(T_0)\right] < 0,$$

i.e. $$\alpha - \rho > 0.$$

However, since by Eq. (17) it is just this conditon that resistances with reversing characteristics do not satisfy, we have the result that the state at point S_2 is always labile. This means however the following:

For two resistances with reversing characteristics connected in series, the state in which both resistances have the same temperature is always labile.

This is a very important result. Since one can view any resistance as being composed of an arbitrary number of partial resistances connected in series, it follows that a uniform temperature distribution must always represent an unstable state in resistances with reversing characteristics. Thus a nonuniform temperature distribution must set in.

§ 10. Which temperature distribution really appears in a resistance with a reversing characteristic, and what is to be expected as the shape of the real current-voltage curve? To answer this question let us imagine the wire to be divided into a number n of partial resistances connected in series. At first we will choose n=2. Thus we consider the case illustrated in Fig. 14. We label the voltage along the lower branch of the characteristic e (up to the current maximum) with e_α. The voltage along the central reversing branch we call e_β, and above the current minimum we call the voltage e_γ. Here e_α, e_β, and e_γ signify voltages across the whole wire; the voltages of the partial resistances are thus half as large. The real state appearing then /432
corresponds to one of the stable intersection points S_1 or S_3. Thus one resistance assumes the voltage $e_\alpha/2$, the other $e_\gamma/2$. The total voltage thus takes the value $(e_\alpha + e_\gamma)/2$. In general the possible total voltages are obtained by taking all combinations of the three elements $e_\alpha/2$, $e_\beta/2$, and $e_\gamma/2$ two at a time; thus

$$e_\alpha;\ \frac{e_\alpha + e_\beta}{2},\ \frac{e_\alpha + e_\gamma}{2},\ \frac{e_\beta + e_\gamma}{2},\ e_\gamma;$$

Only the combination

$$\frac{e_\beta}{2} + \frac{e_\beta}{2} = e_\beta$$

must be left out because of instability. If these five values are plotted as a function of the current then a connected curve is found, which is drawn in Fig. 15.

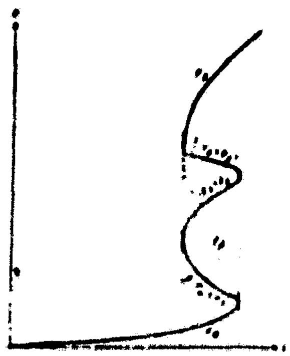

Fig. 15.

Thus the total characteristic of the resistance is composed of the two parts. It is composed of five branches, of which the lowermost and uppermost coincide with the branches e_α and e_γ of the curve applying to a wire at a uniform temperature. For short we will call this curve the "isothermal characteristic" in what is to follow.

In place of the central reversing branch e_β of the isothermal characteristic (dotted in Fig. 15), three new branches have appeared, which all run between the current maximum and current minimum of the isothermal characteristic.

It is easy to convince oneself that, if the voltage E_0 is allowed to run through all values from zero upwards, so that the curve E_0-e in Fig. 14 is displaced upwards parallel to itself from the position $E_0=0$, then the respective intersection points S_1 and S_3 in fact run through current values corresponding to the characteristic of Fig. 15.

The total characteristic can be determined in the same fashion when we imagine the resistance to be divided into n equal parts of size R/n. Then all possible total voltages may be calculated by taking all possible combinations of /433 the three elements e_α/n, e_β/n, and e_γ/n, at a time. The elements e_α/n and e_γ/n may occur several times in a combination. However, e_β/n may not do so, as this would mean lability. Thus the following total voltages are possible:

a) (n+1) combinations without e : $\frac{n e_\alpha}{n}, \frac{(n-1)e_\alpha + e_\gamma}{n}, \frac{(n-2)e_\alpha + 2e_\gamma}{n}, \ldots \frac{2e_\alpha + (n-2)e_\gamma}{n}, \frac{e_\alpha + (n-1)e_\gamma}{n}, \frac{n e_\gamma}{n}.$

b) n combinations with e : $\frac{(n-1)e_\alpha + e_\beta}{n}, \frac{(n-2)e_\alpha + e_\beta + e_\gamma}{n}, \frac{(n-3)e_\alpha + e_\beta + 2e_\gamma}{n}, \ldots \frac{e_\alpha + e_\beta + (n-2)e_\gamma}{n}, \frac{e_\beta + (n-1)e_\gamma}{n}.$

The total characteristic is thus composed of (2n+1) branches, which all run between the current maximum and minimum of the isothermal characteristic and which can easily be drawn. One obtains the first group (combinations without e_β) by taking the ordinates bounded by e_α and e_γ of the isothermal characteristic and dividing them into n pieces. The dividing points of the same order are then connected by (n+1) curves (counting also the e_α and e_γ curves). The terms of the second group (combinations with e_β) differ from the first group by the amount $(e_\beta - e_\alpha)/n$. The curve branches of this group thus divide

the distance $(e_\gamma - e_\alpha)/n$ between two neighboring curves of the first group in the ratio $(e_\beta - e_\alpha) : (e_\gamma - e_\beta)$, i.e., in the same ratio in which the ordinate sections between the e_α and e_γ-curves of the isothermal characteristic are divided by the e_β curve. The (2n+1) branches of the total characteristic then form a connected zigzag line that does not intersect itself. This curve is drawn in Fig. 16 for the case n=10. The first group of branches is labeled with the integers from 0 to 10. The integer gives the number of partial resistances having the voltage e_α and the corresponding temperature T_α. The rest of the resistances have the temperature T_γ. On the branches of the second group lying directly underneath, the number of partial resistances at temperature T_γ is smaller by one, since one resistance has the temperature T_β.

To record experimentally a characteristic of the complicated form of Fig. 16 is only possible with the application of special techniques, which will not be /434
gone into here. We wish, however, to determine the appearance of the characteristic when it is recorded in the usual fashion by connecting the composite resistance to a source of current at voltage E_0 through a series resistance R_v and then gradually reducing the series resistance R_v. What then occurs can be determined by forming the intersection of the line $e=E_0-iR_v$ ("resistance line") with the characteristic in Fig. 16 and then following the movement of the point of intersection as the series resistance is lowered, i.e., the resistance line is rotated in a clockwise sense about its intercept (point D) on the ordinate axis. One then sees the following: at first the intersection point moves rightwards on the lowermost branch of the characteristic e(i), until the resistance line has turned so far that it touches the characteristic at the point P_{10} in the vicinity of the first maximum of e and i. With further rotation the intersection point jumps over to the next branch (9) of the characteristic, moves rightwards on this branch up to the point P_9, then jumps to the branch (8), and so forth, until the branch e_γ is reached. Thus only a few pieces of the characteristic are recorded. If the resistance R_v is allowed to increase so that the resistance line rotates back again, a different outcome results: the intersection point moves back along the branch e_γ to a point somewhat above the uppermost current minimum; then it jumps to the next lower branch (1) of the characteristic e, moves downwards on this branch up to the next minimum of i and e, where it jumps

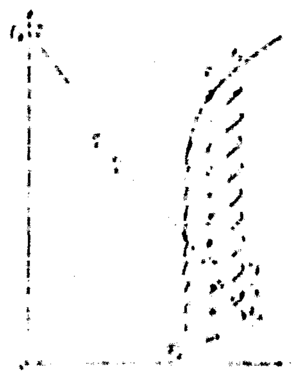

Fig. 16

again to the branch (2). This continues until the branch (10) is reached; the current and voltage then decreases steadily to zero along this branch.

Physically the following occurs. As the voltage gradually increases from zero, the partial resistances all have at first the same temperature T_α , which steadily increases with voltage until the point P_{10} is reached. Then--corresponding to the jump of the intersection point to branch 9 in the diagram-- one of the partial resistances jumps to the higher temperature T_γ , which makes the current fall and the temperature T_α sink correspondingly. By further increase in voltage the temperatures increase steadily again up to the point P_9, whereupon a second resistance takes on the temperature T_γ . This continues until at the point P_0 all of the partial resistances have reached the high temperature T_γ , which from there on, increases uniformly in all of the resistances. On the way back, with decreasing voltage, the analogous process takes place in reverse order, although at lower temperatures, namely temperatures in the vicinity of those temperatures T_{α_2} and T_{γ_2} corresponding to the minimum value i_k of the current. When the voltage was increasing the relevant temperatures lay in the vicinity of the tempeatures T_{α_1} and T_{γ_1} corresponding to the maximum value i_m of i. /435

In order to work out the behavior of a real resistance, which can be thought of as composed of infinitely many partial resistances, let us go to the limit $n = \infty$. Then the individual pieces of the broken zigzag line of Fig. 16 move infinitely close together and become infinitely small. The two zigzag lines then pass over into the vertical straight lines $i = i_m$ and $i = i_k$. The characteristic then assumes the shape shown in Fig. 17, which can be derived from the isothermal characteristic (dashed in Fig. 17) in an obvious fashion.

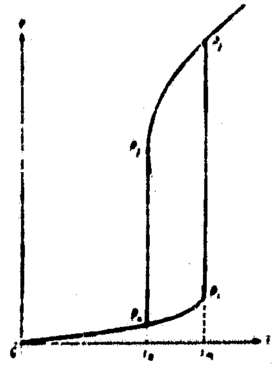

Fig. 17.

/436

The theory thus in fact yields a characteristic similar in shape to Fig. 1, and showing a constant current over a wide range of voltage. However, the theoretical curve has the peculiarity that the constant current value is larger for increasing voltage than for decreasing voltage. The resistance shows hysteresis.

The physical process going on through the vertical stretches of the characteristic is quite similar to that described above for the case of a finite subdivision of the resistance. The only difference here is that now, as a result of the vanishing size of the partial resistances, the alteration in the total length of the hot portion of the wire no longer occurs in finite jumps, but instead

proceeds continuously. In addition the temperatures of the hotter and cooler portions of the wire remain constant as a consequence of the constancy of the current. Thus, with increasing voltage, at the point P_1 (current maximum of the isothermal characteristic) a small piece of the wire (which had a uniform temperature up to this point) suddenly takes on the temperature T_{γ_1} corresponding to the point P_2 ($e = e_\gamma$ (i_m)), while the rest of the wire remains at the temperature T_{α_1} corresponding to the point P_1. If the voltage is allowed to decrease again, the temperature remains at first uniform across the wire--not only up to the point P_2, but beyond that up to the minimum in current of the isothermal characteristic (point P_3), which corresponds to the wire temperature $T_{\gamma_2} < T_{\gamma_1}$. With a further decrease in temperature a small piece of the wire suddenly takes on the appreciably lower temperature $T_{\alpha_2} < T_{\alpha_1}$, corresponding to the point P_4 ($e = e_\alpha$ (i_k)). With decreasing voltage the cold piece grows, while the temperatures remain constant at T_{γ_2} and T_{α_2} respectively, until the point P_4 is reached, where the whole wire has the temperature T_{α_2}. From there down to zero voltage the wire maintains a uniform temperature distribution.

The temperature distribution along the wire on the vertical stretches of the curve thus corresponds to something like that of Fig. 18. The length of the hot piece must, as one easily sees, grow linearly with the voltage.

§ 11. The consideration of § 10 apply rigorously only to a conductor /437
composed of a very large number of discrete pieces. If these pieces are fused to form a continuous wire, an essentially new feature is added, which we have neglected up till now, namely heat conduction through the wire. This prevents the development of discontinuities of temperature along the wire. A temperature distribution such as that of Fig. 18 is therefore in reality excluded, since the infinitely large temperature gradient on the boundary between the hot and cold regions would immediately be transformed into a finite gradient by the thermal conduction. The boundary transition discussed in the previous paragraph is, therefore, not physically admissible. To find the true behavior of our wire we must now investigate the effect of this thermal conduction.[22]

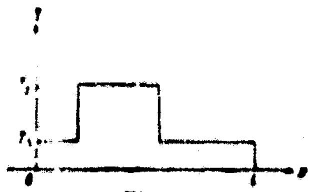

Fig. 18.

[22]We will exclude from consideration the heat developed in the temperature gradient as a consequence of the Thomson effect, since it is always small in comparison with the Joule heat, as an approximate calculation shows.

We begin with the differential equation for the temperature distribution derived above

(6) $$\lambda q \frac{d^2 T}{dx^2} = A(T) - i^2 R(T),$$

where we will from now on understand A(T) and R(T) to refer to a unit length of the wire rather than its total length. We write Eq. (6) in the form:

(6a) $$\frac{d^2 T}{dx^2} = \frac{R(T)}{\lambda q}\left(\frac{A(T)}{R(T)} - i^2\right) = f(T).$$

A single integration yields

$$\left(\frac{dT}{dx}\right)^2 = 2\int f(T)\,dT + \text{const.}$$

We shall measure x from the center of the wire; there symmetry demands that the temperature have a maximum T_m, so that $dT/dx = 0$. This determines the constant of integration, and we obtain

(22) $$\left(\frac{dT}{dx}\right)^2 = 2\int_{T_m}^{T} f(T)\,dT = -2\int_{T}^{T_m} f(T)\,dT.$$ /438

where the integration constant, equal to the maximum temperature T_m, is to be chosen so that

$$x = \pm \frac{l}{2}, \qquad T = T$$

The formal integration, which is obviously easy to carry out, will not be done; instead we will proceed graphically. By Eq. (6a) the function of f(T) contains the expression A(T)/R(T). By Eq. (1) this is nothing other than the square of that current that would have to flow in a wire of infinite length to heat it to an identical temperature T; i.e. A(T)/R(T) is identical with the function $i^2(T)$, and the inverse function is thus identical to $T(i^2)$, which can be taken from the diagram of Fig. 10. This function varies generally like the function $R(i^2)$. In the vicinity of the reversed characteristic, where the current i does not vary much, the variation of these function is similar to that of R(i) and that of the isothermal characteristic e(i). Thus, if the quantity A(T)/R(T) is plotted as abscissa and T is plotted as ordinate, as in the right hand side of Fig. 19, a reversed curve similar to the characteristic e(i) is obtained (solid line). Let us further draw a parallel to the ordinate axis corresponding to the current i, naturally assumed to be constant. On this parallel $A(T)/R(T) = i^2$. The quantity $A(T)/R(T)-i^2$ is then equal to the distance from this line to the

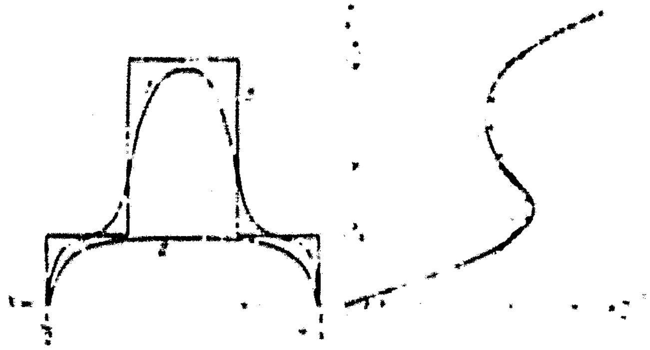

Fig. 19

abscissa calculated for the curve T(A/R), shown by the arrows in Fig. 19. To obtain f(T), this distance must still be multiplied by the factor $R(T)/\lambda q$, i.e., by a quantity which increases with increasing ordinate T. Thus the dashed curve of Fig. 19 results. Here the scale of the abscissa was arbitrarily chosen to make the dashed curve meet the solid curve at the temperature T_β. As a result of the multiplication, the negative maximum of f(T), corresponding to the higher temperature, is relatively enlarged, while the positive maximum at the lower temperature is relatively reduced. The characteristic shape of the curve is, however, not altered. /439

For different currents, i.e. for different positions of the vertical $A/R=i^2$, the dashed curve has different shapes. If the problem is to be solved rigorously, it must be drawn anew for each i. For the following qualitative discussion, however, it is sufficient to use the solid T(A/R)-curve instead of the dashed one, and to carry out the multiplication with $R(T)/\lambda q$ only by use of a suitable scale; that is, this factor is to treated as a constant.

In general the line $A/R = i^2$ intersects the characteristic T(A/R) in three points, whose ordinates we will call T_α, T_β, and T_γ in correspondence with our earlier notation. Then the following results immediately from the differential equation (6a):

1. Since at the maximum of T, $d^2T/dx^2 < 0$ and thus f(T) must be negative, the maximum temperature T_m can only lie between T_β and T_γ or else below T_α. It cannot lie between T_α and T_β. In particular T_m must always be smaller than T_γ.

2. The closer T_m lies to T_γ or T_α, respectively, the shallower is the maximum in T, and the wider apart are the points where the T(x) curve intercepts the abscissa, and thus the greater is the wire length l. Thus with a sufficiently long wire, T_m must lie very near T_γ or T_α.

3. Two different temperature distributions are possible: one, in which the maximum temperature T_m lies near T_{γ}, and the other in which it is near T_{α}.

4. Since f(T) and therefore d^2T/dx^2 is zero at the temperatures T_{α} and /440
T_{β}, the T(x) curve must have inflection points at these temperatures. In the temperature domain between T_{α} and T_{β} ($d^2T/dx^2 > 0$) must be convex towards the x-axis. Everywhere else it must be concave

Here the temperature distribution can already be given in general terms. It is represented on the left hand side of Fig. 19 by curves I and II. The discontinuous line III shows the temperature distribution that would result without taking the thermal conduction into account.

For a more precise determination of the temperature distribution, we refer to Eq. (22). This equation expresses the fact that the square of the temperature gradient is proportional to the area enclosed by the ordinate $A/R=i^2$, the curve T(A/R), and the abscissas corresponding to the temperatures T_m and T.

Let us now consider an excellent special case. Let the current have exactly such a value i* that both areas bounded by the curve T(A/R) and the ordinate $A/R=i^2$ and between T_{γ} and T_{β} on the one hand, or between T_{β} and T_{α} on the other hand, are equal in size. Mathematically speaking then,

(23) $$\int_{T_\alpha}^{T_\gamma} f(T)\, dT = 0$$

More exactly the current should be a very small amount larger than the "normal current" i* defined by this condition. Let us choose T_m very close to T_{γ}; then the temperature will be almost constant over a long stretch of the wire. On the other hand, by Eqs. (22) and (23), $(dT/dx)^2$ will be very near zero at the inflection points at the temperature T_{α}. Thus in this region the temperature is again almost constant over a long stretch of the wire. We thus obtain a temperature distribution similar to that portrayed in Fig. 20. In the center of the wire there is a region of almost constant high temperature, whose value is approximately equal to T_{γ}. At each of the two ends there is a region of lower temperature, also practically constant. Here T is nearly equal to T_{α}. In each case there is a transition region in between, where the temperature falls rather steeply from the higher value to the lower value. There is also a transition region at each end where the temperature falls from the value T_{α} down to the still lower final temperature T_0. In the vicinity of the temperatures /441
T_{γ} and T_{α} we can also describe the temperature variation. Here the f(T) curve can be approximated by a straight line; i.e., f(T) is approximately proportional

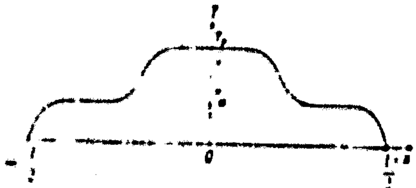

Fig. 20

to ($T-T_\gamma$) and ($T - T_\alpha$), respectively. The constant of proportionality is equal to the slope of the f(T) curve, i.e., equal to the reciprocal of the slope of the T(A/R)-curve in Fig. 19. Then the differential equation (6a) becomes linear and leads to an exponential law, with a constant equal to the square root of the proportionality constant. Thus the T(x) - curve (Fig. 20) approaches the horizontal lines $T = T_\alpha$ and $T = T_\gamma$, respectively, according to the exponential law $e^{-\beta x}$, whose constant β is equal to the square root of the reciprocal slope of the T(A/R) curve at the temperature in question. The length of the transition region (calculated say between the point of steepest slope for the T(x) curve and the point where T has approached the value T_α and T_γ, respectively, to within some definite amount) will be of the order of magnitude of the reciprocal of the constant β. It is hus directly proportional to the square root of the slope of the T(A/R)-curve and thus also to the slope of the characteristic e(i). Now the slope of the T(A/R)-curve (Fig. 19) is not very different at the temperatures T_α and T_γ. If the reversed portion of the characteristic is not very steep, this slope is of the same order of magnitude as the slope of a normally sloping characteristic. Thus it follows that the lengths of the transition regions $\overline{T_0 T_\alpha}$, $\overline{T_\alpha T_\beta}$, and $\overline{T_\beta T_\gamma}$ are not very different from each other. With characteristics that are not too steeply reversed they are of the same order of magnitude as the transition regions at the end of a wire with normally sloping characteristic. /442 The transition from the T_α region to the T_γ region, which is composed of the two regions $\overline{T_\alpha T_\beta}$ and $\overline{T_\beta T_\gamma}$, is thus about twice as long. The steeper is the reversed portion of the characteristic e(i), the longer is the transition and the more smeared out is the boundary between the hot and cold portions of the wire. With a vertical slope in the isothermal characteristic, the transition becomes infinitely long, and the hot and cold parts of the wire are no longer spearate at all.

The temperature distribution of Fig. 20 holds rigorously only for an infinitely long wire. However, it is valid to a satisfactory approximation whenever the wire length is large in comparison with the length of the transition region. Since the wire coordinate x does not appear in the differential equation,

it continues to be satisfied in this case even after a parallel displacement of the transition curve along the x-axis. The length of the hot portion of the wire is thus undetermined at the normal current i*-- but only at this current. Thus

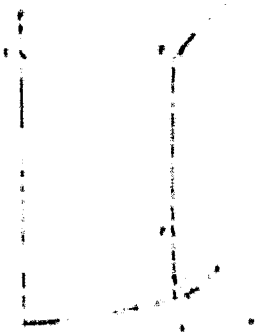

Fig. 21

Thus the average wire temperature can have any arbitrary value between T_α and T_γ, while the voltage can have any value between e_α and e_γ. This yields the wire characteristic immediately. Below the current i* it coincides with the lower branch of the isothermal cnaracteristic; at the normal current i* it rises vertically to the upper branch of the isothermal characteristic, and then coincides with this branch from there on. The lower branch of the isothermal characteristic between the current i* and the current maximum i_m is followed by the wire characteristic, because the temperature distribution, in which the entire wire apart from the ends has the temperature T_α, is always possible[23], /443 as we saw above. Thus the characteristic appears as the heavy solid line in Fig. 21, where in addition the isothermal characteristic is drawn in with dashes.

The true characteristic is thus derived from the isothermal curve in a similar fashion to the way the thermodynamic p,v curve is derived from the "reversing" theoretical Van der Waals curve by replacing the reversed branch by a straight line parallel to the ordinate. The abscissa of the line is chosen that it cuts two pieces of approximately equal area from the reversed curve.

We now wish to investigate the effect of the finite wire length. It will be important only when the length of either the hot portion or the cold portion of the wire becomes comparable to the length of the transition region. In the first case, the maximum temperature T_m will be noticeably smaller than T_γ. Since $(dT/dx)^2$ at $T = T_\alpha$ remains positive by Eq. (22), the i^2 ordinate must be displaced to the right in Fig. 19. Thus the current must become greater than i*. If, conversely, the length of the cold portion of the wire is small, then dT/dx must have an appreciable value at $T = T_\alpha$. Thus the left hand side of Eq. (23) must be greater than zero. Here again from Fig. 19 a rightward dispacement of the i^2-ordinate is

[23] On the other hand, the upper branch of the isothermal characteristic is not part of the wire characteristic for currents i*, since at these currents, d T/dx would be imaginary above t_β, as Figure 19 shows.

necessary. Thus the current is again greater than i*. The corners of the characteristic at the points P_1 and P_2 (Fig. 21) will thus be rounded off, as shown by the thin solid curve in Fig. 21. If at each current the deviations of the voltage in this region from the values e_α and e_γ, respectively, are labeled with δe, it is easily seen that δe is independent of the length of the wire. Since the absolute value of the total voltage is proportional to the wire length, the relative deviation $\delta e/e$ is inversely proportional to the wire length. Thus the larger is the length of the wire, the more exactly the characteristic meets the corners of the characteristic of the infinitely long wire, shown with the heavy line in Fig. 21.

During an experiment the characteristic is traced in the following manner: with increasing voltage the current at first increases along the lower branch up to the current maximum P_0. It then jumps along the "resistance line" P_0P_3, drawn dotted in Fig. 21, until the normal current is reached. From there on the variation follows the characteristic. With decreasing voltage the /444 current follows the characteristic past the point P_3, up to the point P_4 where the resistance line touches the characteristic. Then the current jumps along the resistance line to the somewhat higher value P_5. From then on it proceeds along the lower branch of the characteristic.

At this point we have attained the goal of this section-- to determine the shape of the current-voltage curve to be expected. In the following section we will test the theoretical results against experiment.

III. Experiments

§ 12. In the experiments the same set-up was used as was used in Part I to record the A(T) curves (Apparatus Fig. 2). At first the current-voltage curves were recorded for iron wires of 0.0513 mm diameter. The results at different pressures are represented by the curves of Fig. 22. It is clear that the curves agree in large measure with the theoretically predicted shape, as characterized above in Fig. 21. In particular, the constancy of the current over a wide voltage range is clear, as well as the sharp current maximum on the rising branch of the characteristic, from where the current passes discontinuously over to the constant normal

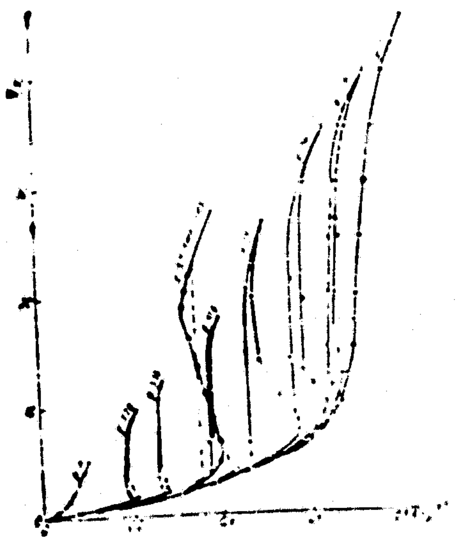

Fig. 22.

value. Very pronounced is the renewed rise of the current on the falling branch of the characteristic shortly before the rising branch is reached again. Even the quantitative agreement is satisfying. In order to test it, the theoretical isothermal characteristic for p = 9 mm from Fig. 10 is drawn in on Fig. 22, /445 together with the measured curves and the actual characteristic that it implies according to the theory. In fact the calculated normal current agrees very accurately with the observed value, when the somewhat different pressures are taken into account. That the characteristics differ not inconsiderably in the ordinate direction, particularly at higher voltages, is not surprising when one considers that, on the one hand the temperature jump at the surface, and thus the shape of the A(T)-curves, depends on the surface treatment of the wire and can therefore be somewhat different for iron than for platinum. On the other hand, the temperature dependence of the resistance of the iron employed here may, as a result of a different degree of purity, very well be different from that of the material used by Somerville, whose data we have used in the calculations.

The observations are particularly favored by the circumstatnce that, as a glance at Fig. 11 will show, the reversed portion of the iron isothermal characteristic falls exactly in the region of temperature where the material is beginning to glow. Thus on the vertical portion of the characteristic the hot portion of the wire glows, while the cool does not. Thus the temperature distribution can be observed directly by eye. In fact on recording the characteristic it is observed that at the moment the current jumps along the resistance line from its maximum value to the normal value, suddenly a short stretch of the wire glows brightly, and then-- without any alteration of the series resistance-- it expands in length with a decrease in brightness, exactly as is to be expected by the theory. If the voltage is then increased, the glowing section grows without altering its brightness, which is a sign that its temperature remains constant,[24] as the theory demands. At those points of the characteristic where the current begins to rise again, the glow has spread out over the entire wire. The sharp boundary between the cold and hot sections and the uniformity of the /446 brightness of the hot sections is expecially striking, particularly at the most

[24]This holds only when the observations are made just after the equilibrium state sets in. In reality, the process goes as follows: At the instant the series resistance is reduced, the current becomes larger as does the brightness of the glowing protion. It is just then that the bright portion of the wire slowly begins to glow, as the current and brightness simultaneously decrease and asymptotically approach their normal values. The last process takes a long time; often 10 or more minutes are required to reach constant value again.

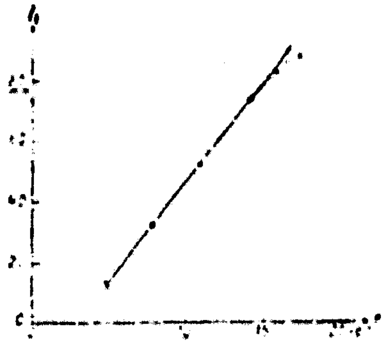

Fig. 23.

favorable pressure (p = 10 mm). Almost directly at the boundary with the cold section the brightness was exactly as great as in the center and the position of the boundary could be conveniently measured to an accuracy of 0.5 mm. In this fashion the length l_1 of the hot section on the vertical part of the characteristic was measured at various voltages with the aid of a reflecting scale. These values are plotted in Fig. 23 as a function of voltage. Exact linearity is obvious, as theory leads us to expect.

At the highest of the applied voltages the wire suffered noticeable lasting alterations from the high temperatures-- most probably as a result of sputtering. Therefore the decreasing branch of the characteristic was always recorded before the increasing branch. In spite of this precaution some of the recorded curves in Fig. 22 show a perceptibly higher normal current on the rising branch than on the falling one. We will return to the origin of this hysteresis later.

Fig. 24 shows the characteristic of two iron wires connected in series. /447
Here a technical resistance was used. It is employed as a series resistance for Nernst incandescent lamps and contains two pairs of iron wires of about 0.07 mm diameter connected in series. Such a resistance was sealed to the pump; it was filled with hydrogen at 5.7 mm pressure, and the outer wall of the glass vessel was maintained at a constant temperature by flowing water. It is noteworthy that two current maxima appear on the characteristic obtained.

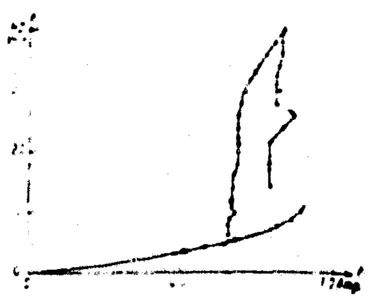

Fig. 24.

This is also to be expected according to the theory. Since after passing the first maximum, only a partial resistance begins to glow, the glow must spread out over its entire length. Thereafter, the current must increase to almost its maximum value before the other partial resistance can be induced to glow. In fact during the experiment only one pair of wires glowed below the upper current

maximum. Above this maximum, both pairs glowed, while on the decreasing branch of the characteristic a portion of each pair was glowing. The hysteresis observed above was particularly strong with the characteristic of Fig. 24.[25]

Further observations were carried out on nickel wires. The results are shown in Fig. 25, where the theoretically constructed isothermal characteristic for p = 9 mm is also entered. These curves lack the pronounced current maximum. Nothing else is to be expected, since the maximum and minimum current are not perceptibly different at all on the isothermal characteristic. The quantitative agreement of the calculated curve with the observed at p = 10 mm is satisfactory in view of the circumstances discussed above. /448

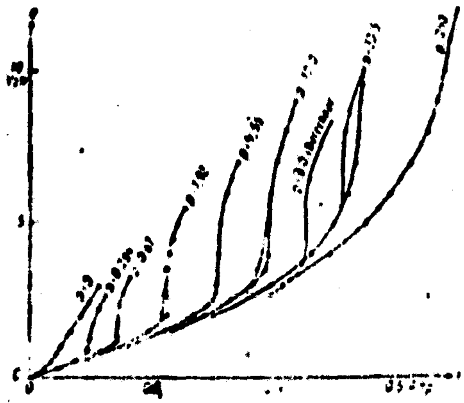

Fig. 25.

Further experiments were performed with copper, silver and tungsten. Similar phenomena often appeared here as well. With copper and silver at a favorable gas pressure the characteristic glow of a short, sharply bounded stretch of wire appeared. However, the temperature rose so high in this case that the wire always burned through very rapidly, so that the complete characteristic could not be recorded. Also with tungsten, the glow appeared on a bounded portion of the wire. It was, in fact, so pronounced that at times the wire glowed white-hot on a stretch less than 1 mm in length, while the rest of the wire glowed only dimly. At this spot the wire rapidly burned through. The

[25]The iron resistances, as they are delivered from industry, have a characteristic similar to that given in Figure 22 for p=210 mm. Pressure, and probably also the type of fill gas, have obviously been chosen to prevent the current maximum from occurring.

theory, which leads us to expect reversing characteristics for these metals, /449
as follows from the intersections of the α - and ρ -curves in Fig. 11, is thus qualitatively confirmed here as well.

The only thing now left is the explanation of the hysteresis observed in the characteristic of the iron wire. If the wire is studied under a microscope, a well-known phenomenon is observed; larger crystallites had formed, which were usually misplaced with respect to each other where they ran together. Thus at these spots the wire cross-section appeared to be reduced and the wire had to glow at a higher temperature there. Because of the cooler places lying in between, the temperature of these spots, which preferentially determine the total voltage across the wire, is not as dependent on the temperature of the neighboring hot areas as in a wire of uniform cross-section. The situation resembles to a certain extent the case of a wire composed of a finite number of short partial resistances whose temperature is independent of one another. We have treated this case in § 10. In fact, the theory leads us to expect the appearance of hysteresis here.

Summary

1. The temperature dependence of the heat loss of thin wires in hydrogen of various pressures was experimentally determined. Two temperature regions appeared, separated by a definite cusp in the curves. Inside these regions the heat loss depended differently on the temperature.

2. With the help of the temperature dependence found above, the current-voltage curve of resistance wires was constructed. It was shown that with resistances having a large positive temperature coefficient, "reversing" characteristics result, on a portion of which the current decreases with increasing voltage.

3. A stability argument showed that a uniform temperature distribution is unstable in wires with a reversing characteristic. It was proved that such a wire separates into two sharply bounded areas of different temperature. Thus /450
a current-voltage curve must appear on which the current is almost constant over a wide range of voltage.

4. The theory was confirmed by experiments on iron and nickel wires, and the hysteresis phenomena appearing here were explained.

The present work was begun at the Radioelectrischen Versuchsanstalt fur Marine und Heer zu Göttingen (The Army and Navy Institute for Radio and Electrical Studies at Göttingen). After the dissolution of this institute,

the work was continued at the Institut for angewandte Electrizität der Universität Göttingen (the Institute for Applied Electricity of the University of Göttingen). I owe a large debt of gratitude to the leader of both institutes, Prof. Dr. M. Reich, for his willing support. I wish to express my thanks here.

Göttingen, in August, 1920.